• pythagorean identity:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

so... $\cos^2\theta = 1 - \sin^2\theta$

1. $4\sin^2\theta - 3 = 0$

$$4\sin^2\theta = 3$$

$$\sin^2\theta = \frac{3}{4}$$

$$\sin\theta = \pm\sqrt{\frac{3}{4}}$$

• don't forget ± when applying a square root

$$\sin\theta = \pm\frac{\sqrt{3}}{2}$$

$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

2. $1 + \sin\theta = 2\cos^2\theta$

$$1 + \sin\theta = 2(1 - \sin^2\theta)$$

• get like terms & set = to zero

$$1 + \sin\theta = 2 - 2\sin^2\theta$$

$$2\sin^2\theta + \sin\theta + 1 - 2 = 0$$

$$2\sin^2\theta + \sin\theta - 1 = 0$$

or $2x^2 + x - 1 = 0$

$$(2x - 1)(x + 1) = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$2\sin\theta - 1 = 0$ $\sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\sin\theta = -1$

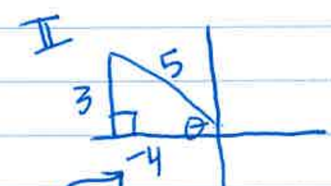
$$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6} \qquad \theta = \frac{3\pi}{2}}$$

3. $\sin 2\theta = 2\sin\theta\cos\theta$ • write formula first!

↓given • Substitute values on the next step!!

$$= 2\left(\frac{3}{5}\right)\left(\quad\right)$$

draw triangle to find $\cos\theta$



$$= \frac{2}{1}\left(\frac{3}{5}\right)\left(\frac{-4}{5}\right)$$

• negative in Quad II

$$= \boxed{\frac{-24}{25}}$$

4. $\tan 165° \rightarrow \tan\frac{330°}{2} = \frac{1 - \cos 330°}{\sin 330°} = \frac{1 - \frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{2\left(1 - \frac{\sqrt{3}}{2}\right)}{2\left(-\frac{1}{2}\right)}$

• multiply by 2 to simplify

• write identity $\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x}$

$x = 330°$

• evaluate as is using unit circle

$$= \frac{2 - \sqrt{3}}{-1}$$

$$= -2 + \sqrt{3}$$

or $\boxed{\sqrt{3} - 2}$

5. $\sin\left(\overset{x}{\frac{\pi}{2}} - \overset{y}{\theta}\right) = \sin x \cos y - \cos x \sin y$

$= \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta$ • Substitute given angle measures & simplify

refer to unit circle

$= (1)(\cos\theta) - (0)(\sin\theta)$

$= \boxed{\cos\theta}$

6. $\dfrac{\csc\theta\cot\theta}{\cos\theta} = \dfrac{\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{1}} = \dfrac{1}{\sin\theta}\cdot\dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\cos\theta} = \dfrac{1}{\sin^2\theta}$

• flip + multiply in numerator

$= \boxed{\dfrac{1}{\csc^2\theta}}$

7. $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{\sin\theta}{1-\cos\theta}$ • get common denominator

$= \dfrac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)} + \dfrac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$ • distribute + FOIL

• middle term cancels
$1+\cos\theta-\cos\theta-\cos^2\theta$
$\rightarrow 1-\cos^2\theta$

$= \dfrac{\sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta} + \dfrac{\sin\theta + \sin\theta\cos\theta}{1-\cos^2\theta}$

$= \dfrac{\sin\theta \cancel{-\sin\theta\cos\theta} + \sin\theta \cancel{+\sin\theta\cos\theta}}{1-\cos^2\theta}$

• substitute pythagorean identity

$= \dfrac{2\sin\theta}{\sin^2\theta}$

$= \dfrac{(2)(\sin\theta)}{(\sin\theta)(\sin\theta)}$

$= \dfrac{2}{\sin\theta} \rightarrow \boxed{2\csc\theta}$

• reciprocal identity

(8.) given: $\sin x = \frac{3}{5}$ Find: $\cos(x+y)$

$\cos y = \frac{12}{13}$ Write formula first! (no values yet)

Positive, acute (Quad I)

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

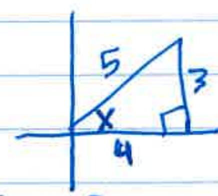
$$= (\quad)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)(\quad)$$

given

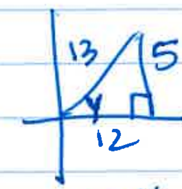
- DO NOT substitute given ratios for the angle measures... keep the $x+y$ as is.

- find other ratios by sketching triangles x and y

- Label angles x & y



$\sin x = \frac{3}{5}$ $\cos x = \frac{4}{5}$



$\cos y = \frac{12}{13}$

$\sin y = \frac{5}{13}$

main work

so... $\cos(x+y) = \cos x \cos y - \sin x \sin y$

$$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$$

$$= \frac{48}{65} - \frac{15}{65}$$

$$= \boxed{\frac{33}{65}}$$

9. $\tan x = \frac{2}{3}$, $\tan y = \frac{1}{2}$ > given

acute & positive

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$= \frac{\frac{2}{3} + \frac{1}{2}}{1 - \frac{2}{3}\cdot\frac{1}{2}}$$

$$= \frac{\frac{4}{6} + \frac{3}{6}}{1 - \frac{2}{6}} \rightarrow \frac{6}{6} - \frac{2}{6}$$

$$= \frac{\frac{7}{6}}{\frac{4}{6}}$$

• keep common denominator, reduce on last step

$$= \frac{7}{\cancel{6}} \cdot \frac{\cancel{6}}{4} = \boxed{\frac{7}{4}}$$

10. $\sin\theta = \frac{\sqrt{7}}{3}$ in Quad II > given

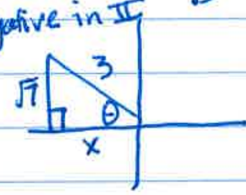
$$\sin 2\theta = 2\sin\theta\cos\theta$$

• write identity then substitute on next step

$$= 2\left(\frac{\sqrt{7}}{3}\right)\left(\frac{-\sqrt{2}}{3}\right)$$

$$= \boxed{\frac{-2\sqrt{14}}{9}}$$

negative in II



$$x^2 + \sqrt{7}^2 = 3^2$$
$$x^2 + 7 = 9$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

11. $\sin 105°$ → • rewrite as

$$\sin\frac{210}{2} = +\sqrt{\frac{1 - \cos 210}{2}}$$

105° is in Quad II so sin = +

Quad III → cos = negative

$$= +\sqrt{\frac{1 - -\frac{\sqrt{3}}{2}}{2}}$$

$$= +\sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$

$$= +\sqrt{\frac{\left(1 + \frac{\sqrt{3}}{2}\right)2}{(2)2}}$$

$$= +\sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= +\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}$$

$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}} \text{ or } \boxed{\frac{1}{2}\sqrt{2+\sqrt{3}}}$$

either form is fine for your answer on the test

Simplify:

(12.) $\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1}$ •get common denominator $\rightarrow \frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta(\cos\theta)}{1(\cos\theta)}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$ •pythagorean identity

$= \frac{1}{\cos\theta}$ •reciprocal identity

$= \boxed{\sec\theta}$

(13.) $\frac{\sec\theta}{\tan\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} = \frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \boxed{\csc\theta}$

•rewrite •flip + multiply •reciprocal identity

(14.) $\frac{1-\sin^2\theta}{2\cos\theta}$ •pythagorean identity $= \frac{\cos^2\theta}{2\cos\theta}$

$= \frac{(\cos\theta)(\cancel{\cos\theta})}{(2)(\cancel{\cos\theta})}$

$= \boxed{\frac{1}{2}\cos\theta}$ or $\frac{\cos\theta}{2}$

(15.) $\frac{\sin 2\theta}{2\cos^2\theta}$ ← •rewrite double angle

$= \frac{2 \cdot \sin\theta \cdot \cos\theta}{2 \cdot \cos\theta \cdot \cos\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \boxed{\tan\theta}$

16. $\cos(90°+\theta) = \cos x \cos y - \sin x \sin y$

$= \cos 90° \cos\theta - \sin 90° \sin\theta$

$= (0)(\cos\theta) - (1)(\sin\theta$

$= \boxed{-\sin\theta}$

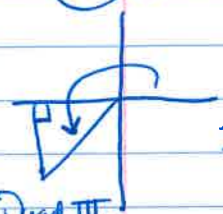
• write identity, then
• substitute values

(similar to #5)

17. $\sin 255°$

• write identity:

$\sin(x+y) = \sin x \cos y + \cos x \sin y$



$\begin{array}{r} 255 \\ -180 \\ \hline 75° \end{array}$ reference angle

Quad III

sin = negative

$\sin(30+45°) = \sin 30° \cos 45° + \cos 30° \sin 45°$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$ ← answer for Quad I (75°)

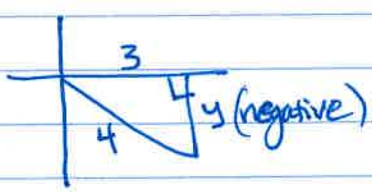
$= \frac{\sqrt{2}+\sqrt{6}}{4}$

$= -\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right) = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$ Quad III solution for 255°

18. given: $\cos\theta = \frac{3}{4}$

Quad IV

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ • write identity



$= \frac{2(\quad)}{1-(\quad)^2}$ • substitute values using a triangle

$= \frac{2\left(\frac{-\sqrt{7}}{3}\right)}{1-\left(-\frac{\sqrt{7}}{3}\right)^2}$

$3^2+y^2=4^2$

$9+y^2=16$

$y^2=7$

$y=-\sqrt{7}$

$= \frac{\frac{2}{1}\cdot\frac{-\sqrt{7}}{3}}{1-\frac{7}{9}} = \frac{-\frac{2\sqrt{7}}{3}}{\frac{2}{9}} = -\frac{\overset{1}{\cancel{2}}\sqrt{7}}{\underset{1}{\cancel{3}}}\cdot\frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} = \boxed{-3\sqrt{7}}$

flip + multiply/cancel

(19.) $\tan^3\theta = 3\tan\theta$ factor & solve

$\tan^3\theta - 3\tan\theta = 0$ gather like terms & set equal to zero

$\tan\theta(\tan^2\theta - 3) = 0$ use ZPP (Zero product property)

$\tan\theta = 0$

$\boxed{\theta = 0, \pi}$

$\tan^2\theta - 3 = 0$

$\tan^2\theta = 3$ apply $\pm\sqrt{}$ to both sides

$\tan\theta = \pm\sqrt{3}$

$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

6 Solutions!!